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REVIEWS OF BOOKS.

A Treatise on Probability. By JOHN MAYNARD KEYNES. London, Macmillan and Co., 1921.—pp. xi, 466.

Those who approach Mr. Keynes's book expecting to find simply a technical treatment of probability will be pleasantly disappointed. It contains that, but it contains much more. Not one, but four, fundamental ideas of importance are constructively developed: that probability is a relation of conclusion to premises like logical relations in general; that probabilities need not be numerically measurable in order to be objective and valid; that the theory of probability admits of formal treatment comparable to that of deduction; and that all induction is properly based on the notion of analogy and the rules of probability. In addition, there is a wealth of critical detail bearing on the formal and the philosophical difficulties of probable inference. The author warns us that "there is much here which is novel, and, being novel, unsifted, inaccurate, or deficient." But the reader will easily be convinced that the method of attack is the right one and the constructive contribution is of the first importance. The subject remains most complex and some problems of theory still undecided. Yet even these matters are set in a clearer light.

The study began, we are told, with an attempt to prove the fundamental theorems of the subject, stated in precise symbolic form, on the hypothesis that probability is a *relation*. "Let our premises consist of any set of propositions h, and our conclusion consist of any set of propositions a, then if a knowledge of h justifies a rational belief in a of degree p, we say that there is a probability relation of degree p between a and h. . . . This will be written $a/h = p$" (p. 4). This conception preserves the objectivity of probability without making it depend on external facts such as frequencies, and separates it as a logical relation from what is merely psychological and subjective without denying that it may vary from person to person and be altered by new knowledge. The probable conclusion, like the certain conclusion, is subject for logical evaluation; is a question of validity, not directly of truth-in-fact.

This extension to probability of the status of logical relations in general is highly satisfactory. But when we ask *what* this relation is, and what are its formal criteria, we find no direct answer in the book. Apparently it is unanalyzable and *sui generis,* even though the certainty which formal logic analyzes is one degree of it. The validity of the probable conclusion can be formally assessed only in the particular cases in which there is already a probability amongst the premises. The *first* probabilities must somehow be apprehended directly: in this respect probability ceases to behave like a logical relation.

Another difficulty concerns a peculiarity of probable as distinguished from certain inference. An ordinary deductive conclusion which is reached by correct modes of inference can never be invalidated by the application to the same premises of any other mode which is also correct. But accepted modes of probable inference give varying results. Whoever applies, for example, the Law of Large Numbers to the probability of four aces in one hand at the next deal will get a result different from that given by the *a priori* method of combinatorial analysis. Thus the probable conclusion seems to be relative not only to premises, but to the rules of inference used. To mitigate this difficulty and others like it, Mr. Keynes so treats the relation a/h as to include in h the rules of inference, with the inevitable result that probabilities become relative not only to differences of acceptable method, but also to 'ignorance of the law.' Is not this to reduce probability once more to the subjective? Add the fact that the basic and 'directly apprehended' probabilities are bound to be subjective (in the sense of being differently apprehended by different persons), and some of the difficulties of the theory become apparent.

The formal criteria of probability are all criteria of comparison. Three rules are here fundamental:

(1) $ab/h < a/h$ unless $b/ah = 1$. That is, the probability of the conclusion 'both a and b' is less than that of a, unless b is always true when h and a are true.

(2) If h_1 is additional evidence which is simple (that is, not divisible into two statements, one of which might be favorable and the other unfavorable) and is relevant to the conclusion a, then a/hh_1 is greater or less than a/h according as h_1 is favorable or unfavorable.

(3) The Principles of Indifference, according to which the probabilities of exhaustive and mutually exclusive alternatives are equal, provided that every part of the evidence is symmetrical with respect to these alternatives.

The precise statement of these rules (we only paraphrase them here) and analysis of the conditions of their application occupy Chapters IV and V. The reappearance of that *bête noire* of clear thinking, the Principle of Indifference, comes as something of a surprise. But the treatment given obviates the worst objections. Whether it obviates them all is a complex and difficult question. In a sense, the Principle of Indifference is an exception to our statement that there are no formal criteria of the basal probabilities.

Although the criteria are comparative, not all probabilities can be compared with one another; and those only can be numerically assessed which are founded on some application of the Principle of Indifference. Sometimes we can affirm one probability to be greater than another when we cannot attach a more determinate numerical measure to either. There will be various series of probabilities such that within each the members stand in relations of greater and less, but not all of these series will consist of probabilities which are numerically assessable. Separate series may have members in common, so that those of series A may be adjudged greater or less than some one member of series B, though such comparison with other members of B may not be possible. Since a series whose members are not in general numerically determinate may have a member or members in common with a series of the opposite sort, probabilities not numerically assessable may still be fixed, on occasion, as greater or less than some definite value. The various series are like different 'paths' between 0 and 1, some of which will intersect, others not.

Where probabilities are relative to premises, not directly to external and only partly known facts such as frequencies, the existence of probabilities which have no numerical measure is indubitable. And one feels that the refusal to reduce all to the arithmetic type is sound and promising for comprehensiveness of treatment. The difficulties of dealing with both numerical and non-numerical cases have been surmounted skilfully and without violence to common sense.

In Part II the fundamental theorems of both certain and probable inference are stated in symbolic form and proved from a set of postulates. There are fourteen definitions and an even greater number of postulates (if each assumption be separately counted). Though the development has not the rigor which we have come to look for in logistic, it is sufficiently precise to be always clear, and positive results are achieved in remarkably brief space. It is postulated that every probability relation a/h is unique. But some will have no numerical value, and this threatens to make trouble when 'sums' and 'products' come in question. For this reason, such expressions are not given a meaning in general, but only in special cases in which the truth of the equations involved is independent of any particular value being assignable. Thus the definition of a sum is given

$$ab/h + a\bar{b}/h = a/h.$$

That is, on the premise h, the probability of 'a and b both' plus the probability of 'a but not b' is *defined as* the probability of a. Just what this sum is to mean when a/h is not numerical is a little puzzling. We are reminded of Jowett's saying that Logic is neither an art nor a science, but a dodge. However, this is a very clever dodge, since it makes possible a uniform symbolic treatment of probabilities, some of which are numerically definite, while others are not. In similar fashion, products are defined only for special cases of the form $a/bh.b/h$:

$$ab/h = a/bh.b/h = b/ah.a/h.$$

This is, of course, the ordinary Multiplicative Axiom: The probability of 'a and b both' is the probability of a when b is present, multiplied by the probability of b.

On such foundations, the symbolic development of fundamental propositions is carried out much further than ever before, and with fewer errors. In view of this, we should do ill to quarrel with the lack of complete rigor. But we may register the wish that some one, if not Mr. Keynes himself, will eventually untangle the complex assumptions of probability and develop a calculus in which symbols do not stand indifferently for relations and numbers, and each expression will have unambiguous and general meaning.

Passing over the subjects of numerical measurement and approximation of probabilities, and much interesting discussion of the application of theorems to problems of different types, we come to Part IV on Induction and Analogy. To the student of Philosophy this is the most important section of the book, since it consciously addresses itself to the problem set by Hume's scepticism. The attempt is to formulate in general terms the procedure which actually underlies the inductive generalizations of science and find the grounds of its justification. The important conclusions are that all induction is fundamentally concerned with analogy, and that pure induction can never establish a generalization, even in the sense of making it highly probable, unless we can find independent grounds for some *a priori* probability.

Analogy consists in the possession of a common property or properties. The propositional function $\phi(x)$ stands for any expression such as 'x is a man,' 'x is vertebrate,' 'x is red.' Hence if the individual objects $a_1, a_2, \ldots, a_n$ are all such that $\phi(a)$ is true of them, this marks an analogy, which may be symbolized by $A(\phi)$. The more comprehensive ϕ, the greater the analogy. The generalizations which induction seeks to establish are always expressions of the form "Whenever $\phi(x)$ is true, $\psi(x)$ also is true." That is, the aim is always to establish a connection between two analogies, $A(\phi)$ and $A(\psi)$, or—what amounts to the same thing—between two parts of a single total analogy which holds for a set of objects or cases. This generalization is symbolized by $g(\phi, \psi)$. But does $g(\phi, \psi)$ formulate the real connection? There may be other properties common to all the objects or cases examined which have been rejected as irrelevant. Let this neglected portion of the actual analogy be represented by $\phi_1(x)$. The problem, then, is this: having examined the objects or cases $a_1 \ldots a_n$, and found two sets of properties, ϕ and ψ, common to all, as well as another set of common properties ϕ_1 which are neglected as irrelevant, to determine the probability that *all other* objects or cases possessed of the properties indicated by ϕ will also possess those indicated by ψ, and to make this probability approach certainty. The more comprehensive ϕ_1 (the greater the neglected analogy), the less this *a priori* probability. Hence the problem of increasing the probability lies mainly in proving the genuine irrelevance of ϕ_1. To put it another way, if we have a given set of objects, their characteristics may be exhaustively classified into (1) those which are common and connected, (2) those which are common but irrelevant, and (3) those which are present in some but absent in others. The first class are those expressed by $g(\phi, \psi)$. The probability that this holds not only for the cases examined, but for all cases, is what is in question. This is the same as the probability that if *all* cases were examined the properties under (2)—and none under (1)—would be disclosed as genuinely irrelevant by being sometimes absent, so that all properties would be divisible into those of class (1) and those of class (3). Assuming, then, that when a connection holds in the first n cases there is some *a priori* probability of its holding in all, this probability is increased when the examination of new cases reduces the common properties rejected as irrelevant. Mr. Keynes calls this 'increasing the negative analogy' and finds in it the chief significance of induction.

By the Inverse Principle, previously proved, if a generalization has an *a priori* probability $\neq 0$, every new instance in which it is verified increases its probability. If, in addition, there is some a priori probability, for each new instance, that the generalization will fail in this case unless it is universally true, then with successive verifications the probability approaches certainty as a limit. Thus the validity of inductive procedure follows from the laws of Part II, with the additional assumption of the a priori probabilities just mentioned.

What is here required is an *a priori* probability which is itself independent of induction. If the existence of an analogy raises any probability, however small, that the same connection will universally hold; and if, for every genuinely novel case, there is a probability, however small, that the generalization will not extend to it unless it also extends to *every* case; then the whole inductive procedure requires no other assumptions except those previously laid down. But Mr. Keynes realizes that the existence of such *a priori* probabilities is not a small question, however small the probability itself. This is an assumption of an entirely different sort from those for probability in general, and has to do with the structure of the natural world. He points out the insufficiency of the usual assumptions of the 'uniformity of nature' and 'universality of law,' and thinks that what is required is the postulation of a 'limitation of the independent variety of the universe,' the notion that the multiplicity of nature is grounded in some lesser multiplicity of 'generator properties.' He is sceptical of our ability to resolve the problem of this assumption "so long as our knowledge of the subject of epistemology is in so disordered and undeveloped a condition as it is at present" (p. 262). But he has made it abundantly clear that what is required for the validity of induction is some independent deduction of the categories. This conclusion is the more impressive because it results from a careful and formal analysis of induction itself.

The "Historical Notes on Induction" and the "Philosophical Applications" are full of matter which is important and of special interest. But these topics are less central and are best left to the reader's examination.

Part V, on "The Foundations of Statistical Inference," affords an excellent critical introduction to this subject. Mr. Keynes is, of course, particularly at home in this field: the best results are summed up, and historical misapprehensions cleared away. He departs to a considerable extent from the main tradition, and these chapters contain much controversial matter.

There is a lengthy bibliography which is of value in itself. The Index is sufficient and accurate.

The thought of the book is so compact, the field which it covers so wide, and the analysis so persistent and penetrating, that this is bound to seem a somewhat formidable volume. But for the same reasons, it merits more than a single reading: very few volumes recently issued so generously reward the careful student.

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La filosofia di G. Locke. Per ARMANDO CARLINI. Firenze, Vallecchi, 1920.—Vol. I, pp. xciv, 287; Vol. II, pp. 379.

These two volumes in the "Collezione il pensiero moderna," edited by E. Codignola, are a welcome and valuable addition to the literature on John Locke. To say this is high praise for any book that undertakes to enlighten us concerning the teachings of a philosopher about whom so much has been written, both during and after his lifetime, as about the great English thinker. It is praise that cannot be withheld from an author who has taken his duties as historian, interpreter, and critic so seriously as has Professor Carlini, and who has done his work so conscientiously, in so scholarly a fashion, with such discriminating judgment and such sympathetic understanding of the subject of his story. We are greatly assisted in our comprehension of the Englishman's thought and of its importance in the history of philosophy by the careful study, made by the Italian writer, of the development of his ideas and of the intellectual movements of the seventeenth century which influenced him either positively or negatively. The great thinkers know their times and take account of the diverse currents of thought that characterize them, and Locke was no exception to the rule; indeed, it is remarkable how well acquainted he was with them, and how deeply interested. To study him, therefore, historically, in the setting of his age, helps us to understand his problem, the obstacles placed in the way of its solution, and the problems which he had to leave to the future to unravel. In this connection, the accounts of the English scientific movement (Vol. I, pp. 53ff.), which was quite virile; of the Port-Royalists (pp. 159ff.); of the Cambridge school (pp. 235ff.); of the controversy over innate principles in England (Culverwell, J. Smith, Whichcote, More, Parker, Cudworth, Cumberland) (Vol. II, pp. 7ff.) are very instructive. The relation of Locke to Bacon and Descartes is treated in an able